Prediction of the Spectral Content of Combination Tone Noise

GORDON F. PICKETT*
Pratt & Whitney Aircraft, Division of United Aircraft Corporation, East Hartford, Conn.

Combination tone noise is generated by a pattern of rotating shock waves in the inlet of turbofan engines when the relative tip speed of the fan blades is supersonic. A method is presented for determining the expected distribution of sound power among the harmonics of engine rotation frequency, based on the spectral analysis of an almost periodic succession of pulses. The spectral distribution of combination tone noise is found to depend on two statistical $-\sigma_a$, the standard deviations of the sequence of shock wave amplitudes; and σ_ε, the standard deviation of the sequence of time intervals between successive shock waves. The spectral distribution of sound power is found to depend more critically on σ_ε than on σ_a. A method of estimating the expected σ_ε for an important type of blade geometry is developed. It is found that blades with straight entrance regions produce higher σ_ε and hence more sound power in the subharmonics of blade passage frequency at the expense of the sound power in the fundamental of blade passage frequency than do blades with curved entrance regions. Typical results of the analytical model and comparisons with experimental data are presented.

Introduction

BECAUSE of the establishment of noise certification requirements of new airplanes, there is a need for accurate noise predictions from new engine designs before experimental data are available. Combination tone noise (also called "buzz-saw" noise and multiple pure tone noise) is a prominent type of noise from current high bypass ratio turbofan engines at takeoff powers. This paper considers the problem of predicting the spectral content of this noise and is applied to estimating the power spectrum from engines with a particular type of fan blades.

Combination tone noise is radiated from the inlet of turbofan engines having fan blades rotating with supersonic tip speeds. Unlike the sound field produced by fans at subsonic operation, where discrete tones are produced at harmonics of blade passing frequency, fans at supersonic tip speeds generate a multiplicity of tones at essentially all integral multiples of engine rotation frequency. Figure 1 illustrates this difference with inlet spectra taken from a full scale research fan rig. This rig is shown in Fig. 2 together with inlet traversing equipment which can be emplaced readily and is capable of mapping the entire inlet field.

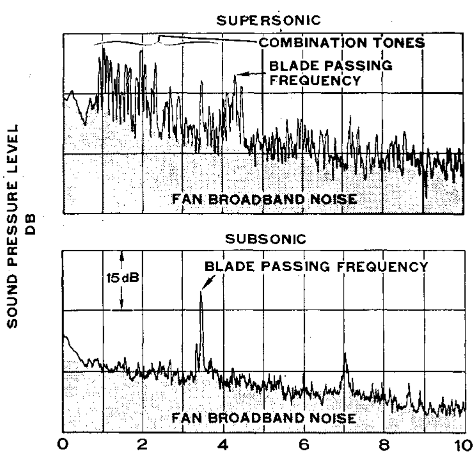

Fig. 1 Fan noise spectra at subsonic and supersonic tip speeds.

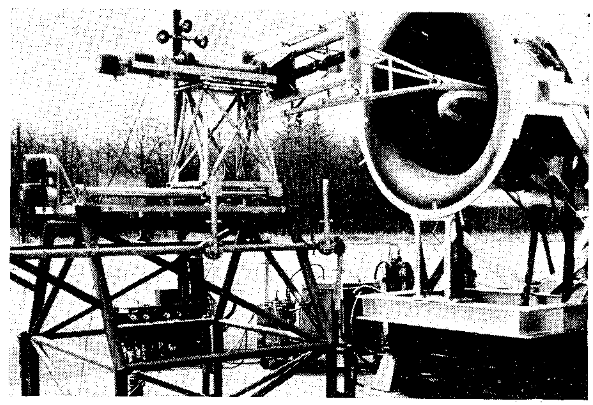

Fig. 2 52-in.-diam fan rig and inlet probe traverse.

The essential features of combination tone noise generation are well established.[1-3] Shock waves are produced at the leading edge of each blade and spiral forward of the fan conveying sound energy out of the inlet to the far field. Figures 3a and 3b show typical pressure-time waveforms at axial locations in the inlet very close to and several chord lengths ahead of the blades. It can be seen that the waveform close to the fan is fairly uniform both in shock amplitude and in the spacing between shocks. The associated spectral analysis confirms this picture with most of the sound power concentrated at blade passing frequency. Further forward of the fan, however, much of the blade-to-blade periodicity is lost and variations in shock amplitude and spacing between shocks is prominent. Since the shocks form a fairly steady but irregular pattern rotating with the fan, the corresponding spectrum is composed of a series of tones at harmonics of shaft rotation frequency.

This loss of blade-to-blade periodicity can be explained on the basis of finite amplitude wave theory.[4] Close to the fan, the intervals between shocks are quite uniform due to the regular spacing of the blades. Some variation in shock amplitude, however, is inevitable because of small manufacturing variations in the incidence angles and other geometric properties of the blades. As the waves spiral forward of the fan, this amplitude variation creates significant interval

Presented as Paper 71-730 at the AIAA/SAE 7th Propulsion Joint Specialist Conference, Salt Lake City, Utah, June 14-18, 1971; submitted June 22, 1971; revision received May 5, 1972. This work was supported in part by the Federal Aviation Administration under Contract DOT-FA69WA-2045.

Index categories: Aerodynamics and Powerplant Noise (Including Sonic Boom); Aircraft Propulsion System Noise.

* Assistant Project Engineer, Acoustics; currently with Rolls Royce (1971) Ltd., Derby, England.

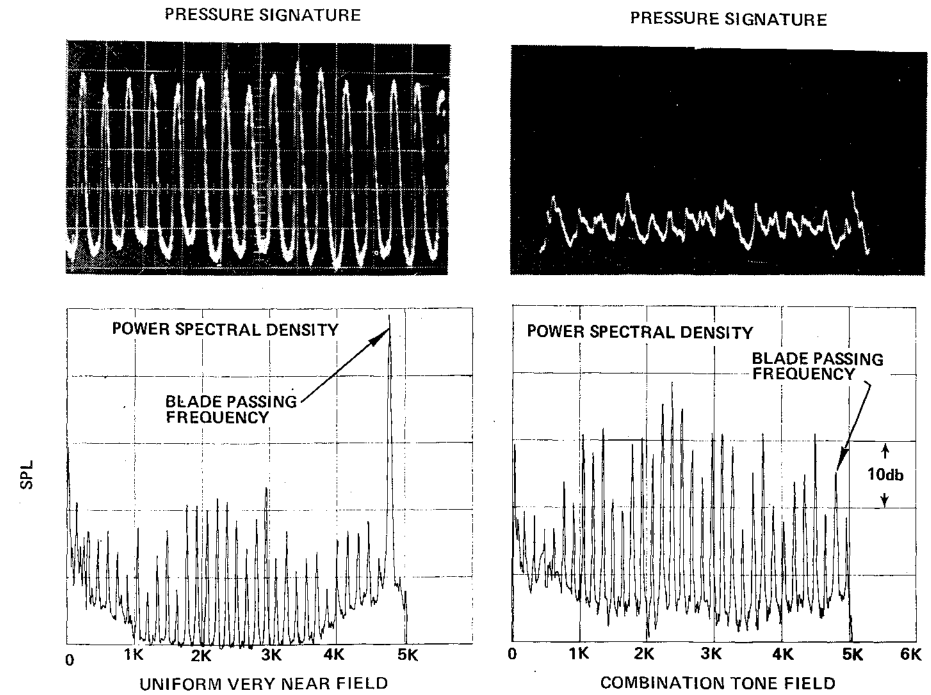

Fig. 3 Waveforms and spectra in combination tone development.

variation because of the influence of shock strength on the rate of propagation. Strong shocks travel faster than relatively weak shocks; thus an initial variation in shock strengths of two consecutive shocks will cause the spacing between these shocks to vary with distance away from the fan. At the same time, both shocks are decaying and they eventually reach a stable situation where the spacing is unchanged with further propagation. Figure 4 gives an example of this effect; a sequence of shocks, as recorded by probing the inlet of a single stage fan rig, evolves into an irregular pattern. A weak shock (denoted by an arrow) preceded and followed by stronger shocks clearly demonstrates the evolution of irregular gaps between these shocks with axial distance ahead of the fan.

A feature of the combination tone spectra in the engine far field is that two fans, although identical in design, produce different spectral signatures. The fact is that each blade is slightly different within well defined manufacturing tolerance bands. When the blades are assembled to produce fans, the small deviations of each fan from design will be different from fan-to-fan. A deterministic prediction procedure for a given design would thus require knowledge of the variations in manufacture of all blades on each fan. Since this is impractical both in advance of and during production, an estimate of the average spectrum for a given fan design is required. This average will not only depend on relevant geometric blade parameters but also on their standard deviations from design.

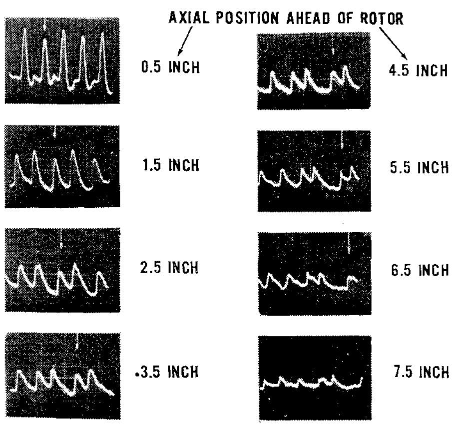

Fig. 4 Change in waveform with distance from rotor.

Before considering the effects of various blade geometries, we will first investigate the spectral properties of a sequence of sawtooth waves similar to those measured in inlets of turbofan engines and rigs. The method used is similar to that used in communication theory and considers a sequence of sawtooth pulses occurring at random time intervals and with random amplitudes which is periodic over the time interval of one revolution. The shock field is assumed to be steady when considered in fan-fixed coordinates.

Spectral Properties of an Irregular Sawtooth Waveform

Consider a fan with B blades rotating E revolutions per second such that the blade tips have a supersonic relative inflow. It is assumed that the shock waves are stationary relative to the fan so that a pressure transducer placed in the inlet will record the passage of B shock waves over the transducer head for each revolution of the fan. If the transducer is placed a few chord lengths upstream of the blades where the shocks are irregularly spaced with variations in amplitude, the signal will consist of B unequally spaced pulses of varied amplitude occurring within a time of $1/E$ sec. This same signal will be repeated every in subsequent interval of $1/E$ sec. Sequences of pulses similar to these have been analyzed in communication theory[5] and the results will be applied here to obtain the expected power spectrum.

If the spectrum (i.e., the Fourier transform) of a typical pulse produced by each shock of unit strength with its following expansion wave and occurring at zero time is $G(\omega)$, the spectrum of the same pulse occurring at time t is $G(\omega)e^{i\omega t}$. If the strengths of the B shocks are denoted by a_n where $0 \leqslant n \leqslant B-1$, and the pulses are regularly spaced, the spectrum of the B pulses for one single revolution of the engine can be written as

$$S(\omega) = G(\omega) \sum_{n=0}^{B-1} a_n \, e^{i\omega n/BE},$$

Because the pulses are not regularly spaced in the time interval of $1/E$ sec, however, the spectrum should allow for variations in the time of arrival of each pulse. Thus

$$S(\omega) = G(\omega) \sum_{n=0}^{B-1} a_n\, e^{(i\omega/BE)(n+\varepsilon_n)},$$

where ε_n/BE represents the time interval by which the nth pulse is delayed. Since the shocks are stationary relative to the fan, the spectrum produced by several revolutions [say $(2M+1)$ for mathematical convenience] is

$$S_M(\omega) = G(\omega) \sum_{m=-M}^{M} \sum_{n=0}^{B-1} a_n\, e^{(i\omega/BE)(n+\varepsilon_n)}\, e^{i\omega m/E} \tag{1}$$

The power spectral density can be obtain from Eq. (1) and is written as

$$PSD = \lim_{M\to\infty} \frac{S_M(\omega)\, S_M{}^*(\omega)}{(2M+1)\,1/E}$$

$$= |G(\omega)|^2 \left\{ \frac{\lim\limits_{M\to\infty} \left[\sum\limits_{M=-M}^{M} e^{im\omega/BE} \right]^2}{(2M+1)\,1/E} \right\}$$

$$\left(\sum_{n=0}^{B-1} a_n\, e^{(i\omega/BE)(n+\varepsilon n)} \right)^2$$

$$= |G(\omega)|^2\, 2\pi E^2\, \delta(\omega - 2\pi kE) \sum_{n=0}^{B-1} \sum_{m=0}^{B-1} a_n a_m$$

$$e^{(i\omega/BE)(n-m+\varepsilon_n-m)} \tag{2}$$

using the result

$$\lim_{M\to\infty} \frac{1}{2M+1} \left[\sum_{m=-M}^{M} e^{im\omega/E} \right]^2 = 2\pi E \delta(\omega - 2\pi kE)$$

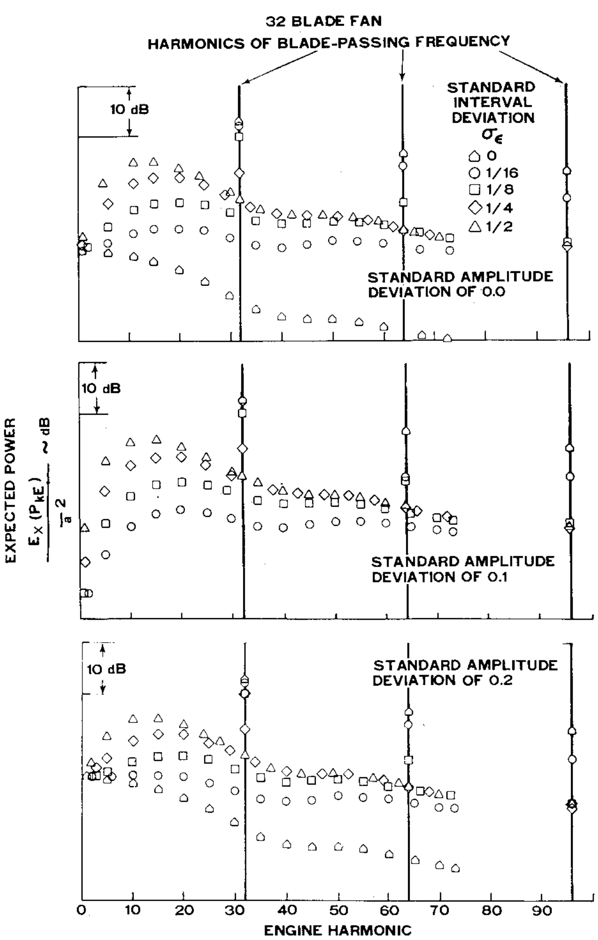

Fig. 5 Expected power in harmonics of engine frequency.

From the properties of the δ-function and the power spectral density, it is seen from Eq. (2) that the power is concentrated at frequencies corresponding to integral multiples of E. The power in the kth harmonic can be written as

$$P_{kE} = |G(2\pi KE)|^2\, 2\pi E^2 \sum_{n=0}^{B-1} \sum_{m=0}^{B-1} a_n a_m\, e^{(2\pi ik/B)(n-m+\varepsilon_n-\varepsilon_m)} \tag{3}$$

Now, the sequence of amplitudes $\{a_n\}$ and the sequence of interval variations $\{\varepsilon_n\}$ are determined by variations in geometry and orientation of the blades and will vary from fan-to-fan. Because of the impracticality of determining values of a_n and ε_n for each fan, a statistical model is required that relates the mean and standard deviation of a_n and ε_n to particular blade shapes and a knowledge of the manufacturing tolerances. In this way an expected power spectrum for a fan design can be obtained.

It is assumed that for a particular fan, the sequence $\{\varepsilon_n\}$ is normally distributed with a mean of zero and a standard deviation of σ_ε. The sequence of values $\{a_n\}$ need not have a specified distribution, but its standard deviation will be given by σ_a. The assumption of independence is not strictly true since the shock intervals are dependent on the relative amplitudes of successive shocks. It was shown in Ref. 2, however, that other factors (such as the variations in expansion regions from blade-to-blade) affect the shock spacing, and for the sake of simplicity the assumption of independence is retained.

If expected values are denoted by brackets, $\langle\ \rangle$, the expected power in the kth harmonic of engine rotation frequency can be obtained from Eq. (3) as

$$Ex(P_{kE}) = \langle P_{kE} \rangle = 2\pi E^2 |G(2\pi kE)|^2$$

$$\left\langle \sum_{n=0}^{B-1} \sum_{m=0}^{B-1} a_n a_m\, e^{(2\pi ik/B)(n-m+\varepsilon_n-\varepsilon_m)} \right\rangle$$

$$= 2\pi E^2\, |G(2\pi kE)|^2$$

$$\left\{ \sum_{n=0}^{B-1} \sum_{m=0}^{B-1} \langle a_n \rangle \langle a_m \rangle\, e^{(2\pi ik/B)(n-m)} \langle e^{2\pi ik\varepsilon_n/B} \rangle \right.$$

$$\left. \langle e^{2\pi ik\varepsilon_m/B} \rangle - \sum_{n=0}^{B-1} \langle a_n \rangle^2 \langle e^{2\pi ik\varepsilon_n/B} \rangle^2 + \sum_{n=0}^{B-1} \langle a_n{}^2 \rangle \right\}$$

$$= 2\pi E^2\, G(2\pi kE)^2 \left\{ \bar{a}^2\, e^{-(2\pi k/B)^2 \sigma_\varepsilon{}^2} \times \right.$$

$$\sum_{n=0}^{B-1} \sum_{m=0}^{B-1} e^{(2\pi ik/B)(n-m)} - B\bar{a}^2\, e^{-(2\pi k/B)^2 \sigma_\varepsilon{}^2} +$$

$$\left. B\bar{a}^2 (1 + \sigma_a{}^2) \right\}$$

using

$$\bar{a}^2 \sigma_a{}^2 = \langle a^2 \rangle - \bar{a}^2$$

$$\langle e^{(\pm 2\pi ik/B)\varepsilon_n} \rangle = e^{-2(\pi k/B)^2 \sigma_\varepsilon{}^2}$$

Thus

$$Ex(P_{kE}) = 2\pi\bar{a}^2\, E^2 B |G(2\pi kE)|^2 \times$$

$$\{1 + \sigma_a{}^2 + e^{-(2\pi k/B)^2 \sigma_\varepsilon{}^2} (B\delta_{kB} - 1)\} \tag{4}$$

where

$$\delta_{kB} = \begin{cases} 1 \text{ if } B \text{ is a factor of } k \\ 0 \text{ otherwise} \end{cases}$$

Since the typical pulse produced by each shock is sawtooth and the time duration of the pulse is taken to be that for a uniform fan (i.e., $1/BE$), the spectrum for a sawtooth of unit amplitude becomes

$$G(\omega) = (BE/\omega^2)[1 - \cos(\omega/BE] + i[(BE/\omega^2)\sin (\omega/BE) - 1/\omega] \tag{5}$$

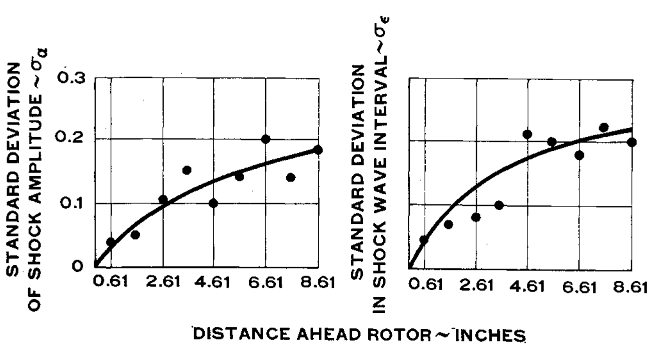

Fig. 6 Shock wave parameters vs distance ahead of experimental fan.

Equation (4) was computed with $G(\omega)$ given by Eq. (5) for various σ_a and σ_ε to yield the expected power as a function or engine harmonic. The results are shown in Figs 5–7 for values of $\sigma_a = 0.0$, 0.1, and 0.2, respectively. A family of curves was generated for values of σ_ε from 0 to 0.5 for the particular case of a 32 blade rotor. The mean amplitude of the shocks was taken to be one; thus, in comparing theory with experimental data, the scaling has to be suitably adjusted. These values of σ_a and σ_ε were chosen so as to cover a measured range obtained by probing the inlet of a rig similar to that shown in Fig. 2. Estimates of σ_a and σ^ε were obtained as a function of distance and results are shown in Fig. 8. As would be expected from the form of the solution (where σ_ε is in the argument of an exponential term), the power spectrum is critically dependent on the standard deviation in shock spacing and only slightly upon the standard deviation of shock amplitude. It should be observed that for all σ_a presented, more power is concentrated in the combination tones at the expense of blade passing frequency (and its harmonics) as σ_ε is increased. In particular, when $\sigma_\varepsilon = 0.5$, the expected power at harmonics of blade passing frequency is indistinguishable from the expected power in the surrounding combination tones. It can also be seen that the expected power in the combination tones initially increases with engine harmonic, attaining maximum values about half way between zero and blade passing frequency. The expected power then decreases with increasing engine harmonic.

These results are consistent with measured data where it is found that the combination tones are most prominent up to blade passing frequency. Occasionally, some tones containing significant amounts of sound power are seen above blade passing frequency; usually, however, when they occur, their contribution to the over all combination tone sound power level is small.

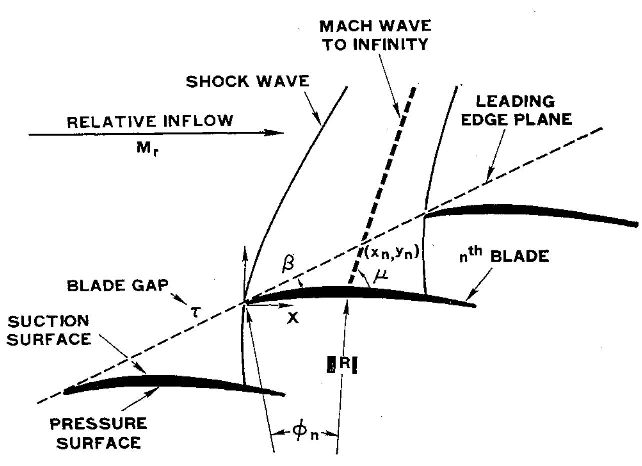

Fig. 7 Compressor blade geometry.

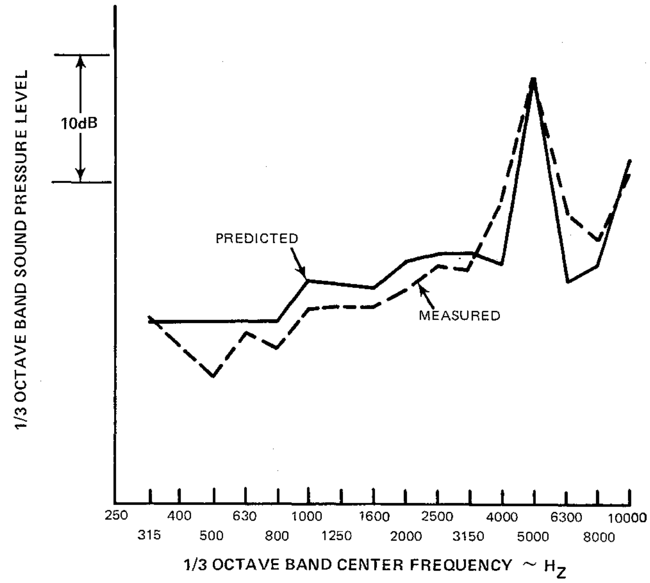

Fig. 8 Comparison of combination tone distributions.

Having established the fact that the variance in shock interval (σ_ε) is the critical parameter affecting the spectral content of combination tone noise, it remains to estimate values of this parameter in the shock far field for specific fan blade geometries. Most fans designed for supersonic tip speed operation are designed with nearly flat suction surfaces. If the suction surface is curved, the flow expands after the leading edge shock and the pressure upstream of the passage shock (i.e., the shock between consecutive blades) decreases resulting in a stronger shock and a loss in fan efficiency. For the purposes of this paper, it is instructive to consider the evaluation of σ_ε for blades with curved suction surfaces, because there is a simple closed form expression for this case. The result obtained can be used to indicate trends for blades with flat suction surfaces.

Estimate of σ_ε for Blades with Curved Suction Surfaces

Consider the flow about a representative compressor blade as shown in Fig. 9. R denotes the suction surface radius of curvature, τ the blade gap, β the angle produced by the freestream flow direction and the leading edge plane of the compressor blades, and ϕ is an angle that determines a point on the nth blade from which a Mach wave is produced that travels unimpeded out to the free stream region. It is, of course, this Mach wave that determines the freestream flow.[6]

It is assumed that with an established freestream flow, individual blades are randomly perturbed by pivoting about their leading edges so as to leave the freestream flow direction unaltered. The freestream Mach waves from each blade will

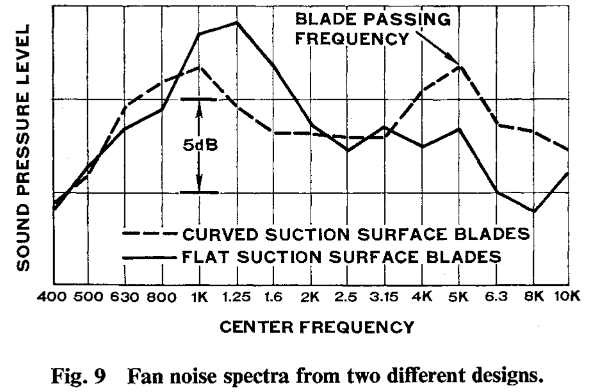

Fig. 9 Fan noise spectra from two different designs.

also be perturbed causing the spacing between successive waves to become irregular. A sequence of angles $\{\phi_n\}$ for $0 \leqslant n \leqslant B\text{-}1$ will be produced that determines the points on each blade from which the freestream Mach wave is produced. It is possible to show, from finite amplitude wave theory, that in the far field the shock waves that are produced at the leading edge of each blade will asymptotically locate themselves midway between the freestream Mach waves. This, the irregularity of spacing between the shock waves is exactly presented, in the far field, by the irregularity in spacing between the freestream Mach waves.

The spacing between freestream Mach waves can be found by considering their intersections with the blade leading edge plane. In coordinates indicated for the zeroth blade in Fig. 9, the intersection of the freestream Mach wave with the leading edge plane can be found for each blade to be (x_n, y_n) for $0 < n < B\text{-}1$, where

$$x_n = n\tau + \left(\frac{R\{\cos(\mu - \phi_n) - \cos\mu\}}{\sin(\mu - \beta)}\right)\cos\beta$$

$$y_n = n\tau + \left(\frac{R\{\cos(\mu - \phi_n) - \cos\mu\}}{\sin(\mu - \beta)}\right)\sin\beta$$

The distance between the nth and $(n+1)$th intersection is

$$\begin{aligned} g_n &= [(x_{n+1} - x_n)^2 + (y_{n+1} - y_n)^2]^{1/2} \\ &= \tau + [R/\sin(\mu - \beta)]\{\cos(\mu - \phi_{n+1}) - \cos(\mu - \phi_n)\} \\ &= \tau + [R\sin\mu/\sin(\mu - \beta)](\phi_{n+1} - \phi_n) \end{aligned}$$

since in all practical cases, the ϕ_n are very small.

The g_n are related to the percentage deviations of the shocks from their regular ε_n by

$$\varepsilon_n = (g_n - \tau)/\tau$$

and thus the standard deviation of the sequence ε_n is found to be

$$\sigma_\varepsilon = (2)^{1/2}(R/\tau)[\sin\mu/\sin(\mu - \beta)]\sigma_\phi \tag{6}$$

where σ_ϕ is the standard deviation of the sequence ϕ_n.

In order to test these ideas, a 28-in-diam fan rig with 32 curved entrance region blades was installed, giving tip values of $R/\tau = 3.6$ and $\beta = 30°$. It was operated at a relative tip Mach number of 1.2 and the combination tone noise power in each third octave band was determined from readings of several far field inlet microphones.

It was determined that the standard deviation σ_ϕ of the blade angle of incidence was $\frac{1}{4}°$. Thus, with $\mu = 56°$,

$$\sigma_\varepsilon = (2)^{1/2}(3.6\sin 56°/\sin 26°)\pi/720 \approx 0.042$$

Since the distribution of power over the entire frequency spectrum is only weakly dependent on σ_a, a value of 0.2 measured a few chord lengths ahead of the fan was assumed to be sufficiently accurate for calculating the far field distribution. The calculated values were summed on a third octave band basis and scaled so that the predicted over-all power agreed with the measured data. Figure 8 shows the comparison.

The essential feature of Eq. (6) is that, as R/τ increases (i.e., as the curved suction surface of the blade tends to a flat suction surface), the parameter σ_ε increases, and more sound power is concentrated in the combination tones at the expense of blade passing frequency harmonics. Figure 9 confirms this trend, comparing $\frac{1}{3}$ octave band spectra of a 60° inlet angle from two dissimilar fans; one contains blades with curved suction surfaces and the other has blades with essentially flat suction surfaces. It is seen that the spectra from the fan with flat suction surface blades has more sound power concentrated in combination tones at the expense of blade passing frequency than is the case for the fan with curved suction surface blades.

It should be noted that the method presented here for determining σ_ε breaks down when R/τ gets too large because the variations in stagger angle will result in having some Mach waves that would have travelled to the freestream either being interrupted by the following blade or becoming part of the so called head shock structure. Since, as mentioned earlier, most supersonic blades are designed with essentially flat suction surfaces, this case is important and is discussed more fully in Ref. 4. Basically, when some of the blades produce no Mach wave that travels to the freestream, the shocks from these blades merge with the shocks immediately following to produce a number of shocks remaining at the inlet that is less than the number of blades. The distribution of power among the combination tones is them determined by the variance in interval between the remaining shocks. The inlet of a rig containing flat suction surface fan blades was probed with a microphone and the data, shown in Fig. 10, clearly illustrate the shock merging phenomenon.

Conclusions

The essential features of combination tone noise generation have been described and used to formulate an analysis that predicts the expected power distribution among the harmonics of shaft rotation frequency for a specific blade geometry. The main conclusions can be summarized as follows.

1) Combination tones are generated as a direct result of small amplitude variations in the blade-attached shocks due to normal manufacturing tolerances. These shocks propagate at different speeds creating shock interval variations forward of the fan.

2) The expected power spectrum of the shock field depends critically upon variations of the intervals between shocks and only slightly upon variations in shock strength.

3) Since fan irregularities due to manufacturing tolerances are not exactly duplicated from fan-to-fan, the combination tone noise spectra from fans of the same design will be different. Accordingly, the analysis must of necessity be concerned with the "expected" spectrum of an "average" fan of a given type. Measured individual fan spectra will be distributed about such an average.

As the suction surface of fan blades becomes flatter, the standard deviation of shock spacing increases and on average there is a shift of power out of blade passing frequency harmonics into the combination tones.

5) For blade designs where there is no shock merging, most of the combination tone sound power is concentrated in the tones up to blade passing frequency with the expected peak at frequencies about one-half of blade passing frequency.

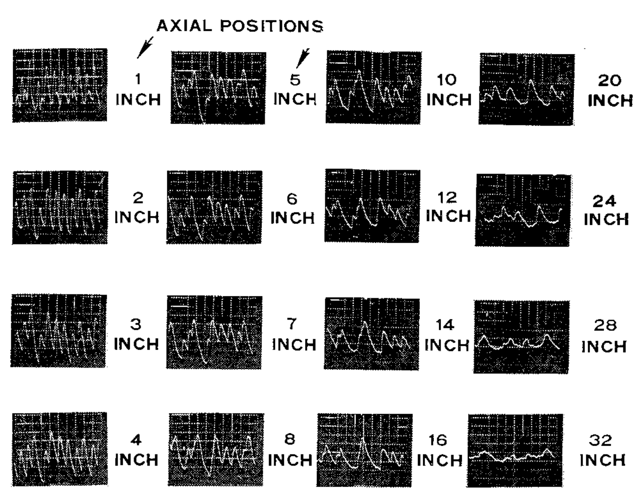

Fig. 10 Pressure-time waveforms illustrating shock merging.

References

[1] Kester, J. D., "Generation and Suppression of Combination Tone Noise From Turbofan Engines," Paper 19, *Proceedings AGARD Fluid Dynamics Panel*, Saint-Louis, France, May 1969.

[2] Sofrin, T. G. and Pickett, G. F., "Multiple Pure Tone Noise Generated by Fans at Supersonic Tip Speeds," *International Symposium on the Fluid Mechanics and Design of Turbomachinery*, Pennsylvania State Univ., Sept. 1970, to be published.

[3] Kester, J. D. and Pickett, G. F., "Application of Theoretical Acoustics to the Reduction of Jet Engine Noise," *Journal of Physics Ser. D; Applied Physics*, Vol. 5, 1972, pp. 12–27.

[4] Burdsall, E. A. and Urban, R. H., "Fan-Compressor Noise: Predictions, Research, and Reduction Studies," FAA-RD-71-73, Feb. 1971, Federal Aviation Administration, Washington, D.C.

[5] MacFarlane, G. G., "On the Energy Spectrum of an Almost Periodic Succession of Pulses," *Proceedings of the Institute of Radio Engineers*, Vol. 37, Oct. 1949, pp. 1139–1143.

[6] Ferri, A., "Aerodynamic Properties of Supersonic Compressors," *Aerodynamics of Turbines and Compressors*, edited by W. R. Hawthorne, Princeton University Press, Princeton, N.J., 1964, Chap. 2.